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MODIFIED VAN DER PAUW METHOD OF MEASURING THE ELECTRICAL CONDUCTIVITY TENSOR OF ANISOTROPIC SEMICONDUCTOR FILMS

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For a solution of the boundary electrodynamic problems, the paper proposes the original method of measuring the specific conductivity tensor components of anisotropic semiconductor square-shaped films. Ohmic and sensitive contacts are placed on the perimeter of the semiconductor film, in accordance with the Van der Pauw method more often used in practice. The derived equations are given in the form of polynomial dependencies of the anisotropy parameter. CdSb *and* $CdAs_2$ *single crystal semiconductors are used in the experiment.*

Keywords: anisotropic semiconductor, specific conductivity, Van der Pauw method.

INTRODUCTION

Materials with different electrophysical properties dependent on the crystallographic direction, *i.e.* anisotropic materials, are becoming increasingly utilized at the current stage of development of semiconductor technology [1, 2]. Their spread is however restrained by the necessity of accounting for the electric field distribution which is more complicated than in isotropic semiconductors and by difficult calculations of kinetic parameters.

Most of researchers do not regard the tensor quantity of electrical conductivity of anisotropic semiconductors (A_2B_5 single crystals) when measuring it by the Van der Pauw method. In works [2–6] the authors take the tensor of electrical conductivity into account indirectly, *i.e.* they note that obtained the average value of this parameter. Therefore, it is necessary to create and approve a non-destructive technique of the experimental identification of the electrical conductivity tensor in compliance with the Van der Pauw method that employs a four-point probe placed around the perimeter of the sample [7, 8].

Let us consider relatively common layouts for the ohmic and sensitive contacts deposited onto the square-shaped semiconductor films [9, 10].

TECHNIQUE 1

Figure 1 depicts the square-shaped semiconductor film cut off along the principal axes of the specific electrical conductivity tensor σ that can be written as

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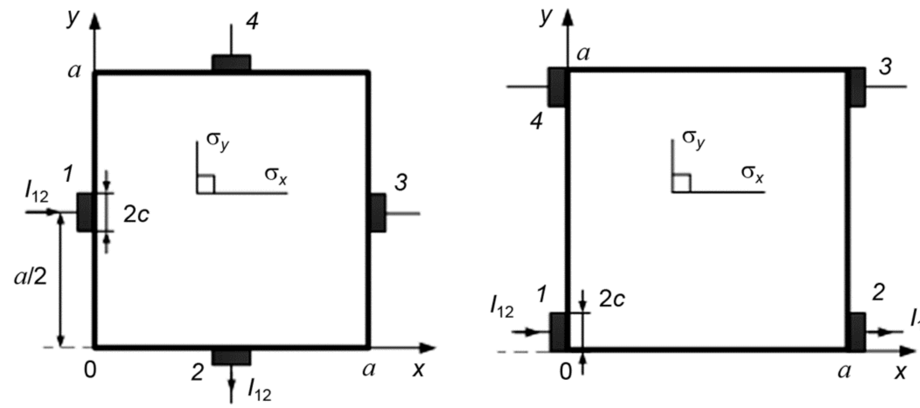

Fig. 1. The contact arrangement in the centre (*a*) and corners (*b*) of the lateral surface of the sample.

$$\hat{\sigma} = \begin{bmatrix} \sigma_x & 0 \\ 0 & \sigma_y \end{bmatrix}, \tag{1}$$

where σ_x, σ_y are specific conductivities on *Ox*- and *Oy*-axes, respectively.

The geometry of the semiconductor film includes *a* length and width and *d* thickness ($d << a$). In Fig. 1*a*, the ohmic and sensitive contacts are placed along the perimeter of the sample, in the centre of the lateral surfaces and have the similar width 2*c*.

When measuring the electrical conductivity tensor, the direct current I_{12} flows through contacts *1* and *2* placed on the neighbor sides (Fig. 1*a*). The voltage is measured between the contact pairs *1*, *3* and *2*, *4*. The theoretical values of the voltage between these contact pairs are as follows [11, 12]:

$$U_{13} = \frac{I_{12}}{d}\frac{1}{\sigma_x} L_{13}(\gamma, c/a), \tag{2}$$

$$L_{13}(\gamma, c/a) = \frac{1}{2} + \frac{1}{4\pi^3\gamma}\sum_{k=1}^{\infty}\left\{\frac{\sin^2(2\pi k\, c/a)}{k^3 (c/a)^2}\frac{\cosh(2\gamma\pi k) - 1}{\sinh(2\gamma\pi k)}\right\},\ \gamma = \sqrt{\frac{\sigma_y}{\sigma_x}}, \tag{3}$$

$$U_{24} = \frac{I_{12}}{d}\frac{1}{\sigma_y} L_{24}(\gamma, c/a), \tag{4}$$

$$L_{24}(\gamma, c/a) = \frac{1}{2} + \frac{\gamma}{4\pi^3}\sum_{k=1}^{\infty}\left\{\frac{\sin^2(2\pi k\, c/a)}{k^3 (c/a)^2}\frac{\cosh(2\pi k/\gamma) - 1}{\sinh(2\pi k/\gamma)}\right\}. \tag{5}$$

Using Eqs. (2)–(5), we get the function

$$f_1(\gamma, c/a) = \frac{U_{13}(\gamma, c/a)}{U_{24}(\gamma, c/a)} = \gamma^2 \frac{L_{13}(\gamma, c/a)}{L_{24}(\gamma, c/a)}. \tag{6}$$

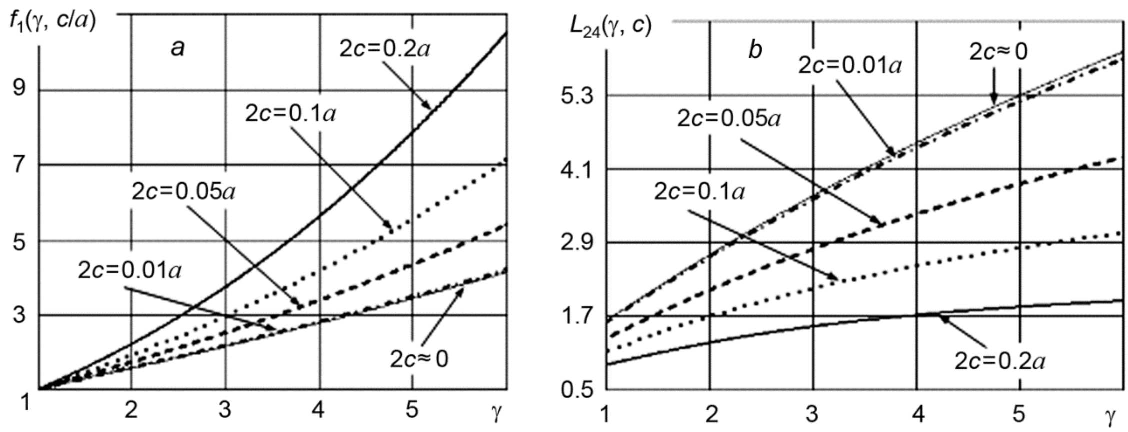

Fig. 2. Dependencies between $f_1(\gamma, c/a)$ and γ (*a*) and $L_{24}(\gamma, c/a)$ and γ (*b*) at different contact width 2*c*.

According to calculations, 100 terms of series is sufficient to achieve a 1% measurement error in Eqs. (3) and (5) in practice cases of $2c/a \le 0.01$, $\gamma \le 10$. In these cases, the following approximate equation can be applied to $2c/a \cong 0$ contact points [9, 10] in Eqns (3) and (5):

$$\frac{\sin\left(2\pi kc/a\right)}{2\pi k(c/a)} \approx 1. \tag{7}$$

In the case of the low ohmic contacts ($2c/a \le 0.01$), it is suitable to utilize the obtained polynomial dependencies $f_1(\gamma)$ and $L_{24}(\gamma)$ with the certainty factor $R^2 > 0.99$:

$$f_1\left(\gamma\right) = 1.87\cdot 10^{-2}\cdot\gamma^2 + 0.502\cdot\gamma + 0.496, \tag{8}$$

$$L_{24}\left(\gamma\right) = 3.28\cdot 10^{-3}\cdot\gamma^3 - 8.04\cdot 10^{-2}\cdot\gamma^2 + 1.28\cdot\gamma + 0.405. \tag{9}$$

In Fig. 2 the dependencies of $f_1(\gamma, c/a)$ and $L_{24}(\gamma, c/a)$ on the anisotropic parameter γ are presented for different values of the contact width. In these calculations the anisotropic parameter γ ranges from 1 to 6. This is because at $0 < \gamma < 1$, the dependence between $f_1(\gamma, c/a)$ and the anisotropic parameter γ is not visible within the selected scale level, whereas $\gamma > 6$ in semiconductors used in practice, is very rare [1, 2, 13]. The analysis of Eqns (3), (5) and Fig. 2 shows that the use of the approximate equation (7) (with a 1% error) is advisable when the relative size of electrode is $2c/a \le 0.01$.

The analysis of Eqns (2)–(6) and dependencies presented in Fig. 2, allows us to suggest the measurement technique 1 for the tensor components of electrical conductivity.

1) When current I_{12} passes through contacts *1* and *2,* U_{13} and U_{24} voltages are measured. Then the relationship $U_{13}/U_{24} = f_1^{\exp}\left(\gamma, c/a\right)$ is found. If f_1 parameter is found to be less than 1, the sample should be rotated at a 90-degree angle, with the respective change in the numbers of the ohmic and sensitive contacts, and the voltages should be measured again.

2) Assuming that $f_1^{\exp}\left(\gamma, c/a\right) = f_1^{\text{theor}}\left(\gamma, c/a\right)$, measure the anisotropic parameter γ using either the constructed dependencies or Eqns (6) or (8)–(11).

3) Calculate $L_{24}(\gamma, c/a)$ multiplier using the obtained anisotropic parameter γ. Knowing the values of I_{12}, U_{24} and L_{24}, find the tensor components σ_x and σ_y of electrical conductivity:

$$\sigma_y = \frac{I_{12}}{U_{24}} \frac{1}{d} L_{24}, \; \sigma_x = \sigma_y / \gamma^2 . \tag{10}$$

TECHNIQUE 2

Let us consider the arrangement for the ohmic contacts $2c$ wide at the corners of the lateral surfaces of the crystal oriented along the principal axes of the specific conductivity tensor (Eq. (1)). The layout used for these contacts is illustrated in Fig. 1*b*.

Initially, direct current I_{12} passes through the current contacts *1* and *2* positioned on the opposite sides. Using the electric potential equation [11, 12], we find the voltage between sensitive contacts *3* and *4* (Fig. 1*b*):

$$U_{34} = \frac{I_{12}}{d} \frac{1}{\sigma_x} L_{34}\left(\gamma, c/a\right), \tag{11}$$

$$L_{34}\left(\gamma, c/a\right) = 1 + \frac{4}{\gamma} \sum_{n=1}^{\infty} \left\{ \left(\frac{\sin\left(\pi n c/a\right)}{\pi n c/a} \right)^2 \frac{\cos\left(\pi n c/a\right)}{\pi n} \cos\left[\pi n \left(1 - \frac{c}{a}\right) \right] \tanh\left(\gamma \frac{\pi n}{2} \right) \right\}. \tag{12}$$

In order to measure the two tensor components in Eq. (1), the direct current I_{14} should pass through contacts *1* and *4* and the voltage U_{23} should release from contacts *2* and *3* (Fig. 1*b*). In this case, the voltage takes the form [11, 12]:

$$U_{23} = \frac{I_{14}}{d} \frac{1}{\sigma_y} L_{23}\left(\gamma, c/a\right), \tag{13}$$

$$L_{23}\left(\gamma, c/a\right) = \frac{8\gamma}{\pi} \sum_{n=1,\,3,\,\ldots} \left\{ \left[\frac{\sin\left(\pi n c/a\right)}{\pi n c/a} \right]^2 \frac{\sin^2\left[\pi n \left(c/a - 0.5\right)\right]}{n \cdot \sinh\left(\gamma \pi n\right)} \right\}. \tag{14}$$

Based on Eqns (11)–(14), the voltage ratio function is defined as follows

$$f_2\left(\gamma, c/a\right) = \frac{U_{34}}{U_{23}} = \gamma^2 \frac{I_{12}}{I_{14}} \frac{L_{34}\left(\gamma, c/a\right)}{L_{23}\left(\gamma, c/a\right)}. \tag{15}$$

The linear polynomial dependences $f_2(\gamma)$ and $L_{34}(\gamma)$ ($R^2 > 0.99$) are suggested for the contact points ($2c \le 0.05a$ condition is sufficient for this arrangement):

$$\ln\left[f_2(\gamma)\right] = -1.44 \cdot 10^{-2} \cdot \gamma^4 + 0.245 \cdot \gamma^3 - 1.57 \cdot \gamma^2 + 7.91 \cdot \gamma - 6.53, \tag{16}$$

$$L_{34}(\gamma) = 4.24 \cdot 10^{-4} \cdot \gamma^5 - 9.80 \cdot 10^{-3} \cdot \gamma^4 + 9.16 \cdot 10^{-2} \cdot \gamma^3 - 0.444 \cdot \gamma^2 + 1.18 \cdot \gamma - 0.613. \tag{17}$$

The dependences of function $f_2(\gamma, c/a)$ on the anisotropic parameter γ are plotted in Fig. 3 for the different contact width $2c$ at $I_{12} = I_{14}$. And Fig. 4 shows the dependences between the multiplier $L_{34}(\gamma, c/a)$ and the anisotropic parameter γ for the different contact width $2c$. According to Fig. 1*b*, and the analysis of these dependencies, the approximate equation (7) (with a 1% error) can be used for the measuring diagram, when $2c \le 0.05a$.

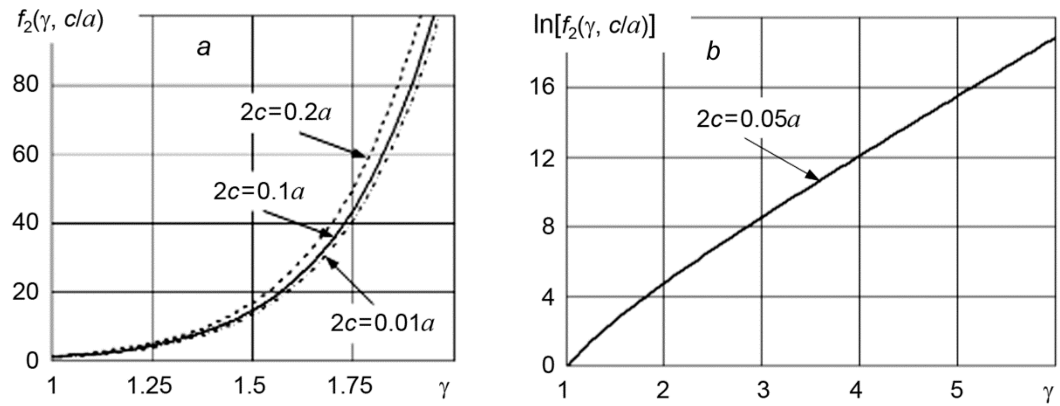

Fig. 3. Dependences between $f_2(\gamma, c/a)$ and anisotropic parameter γ.

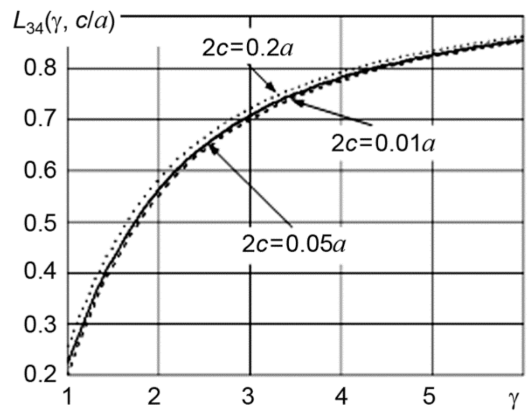

Fig. 4. Dependences between $L_{34}(\gamma, c/a)$ multiplier and anisotropic parameter γ at several values of the relative contact width $2c/a$.

The analysis of Eqns (11)–(17) allows us to suggest the measurement technique 2 for the specific conductivity tensor components of anisotropic semiconductor square-shaped films with the contacts arranged at the corners:

1) When the current I_{12} flows across contacts *1* and *2* positioned on the opposite sides of the square-shaped films, the voltage U_{34} between contacts *3* and *4* (Fig. 1*b*) should be measured. Then, when the current $I_{14} = I_{12}$ flows through contacts *1* and *4* positioned on one side of the crystal, the voltage U_{23} is measured. The ratio between the obtained voltages $U_{34}/U_{23} = f_2^{\exp}(\gamma, c/a)$ is then calculated.

2) Assuming that $f_1^{\exp}(\gamma, c/a) = f_1^{\text{theor}}(\gamma, c/a)$ and using the diagrams above or Eq. (16), determine the anisotropic parameter γ.

3) Based on the determined anisotropic parameter γ, calculate $L_{34}(\gamma, c/a)$ multiplier using either Eq. (17) or (12). Knowing I_{12}, U_{34} and L_{34} values derive σ_x and σ_y tensor components from Eq. (11):

$$\sigma_x = \frac{I_{12}}{U_{34}} \frac{1}{d} L_{34}, \ \sigma_y = \sigma_x \gamma^2 . \tag{18}$$

TABLE 1. Geometrical Dimensions of Samples and Contacts.

Samples	Linear size a, mm	Thickness d, mm	Contact width $2c$, mm	$<f^{ex}>$	γ	L_{24}	σ_x, $Ohm^{-1} \cdot m^{-1}$	σ_y, $Ohm^{-1} \cdot m^{-1}$
Ge	15	0.6	1.2	1	1	0.977	12.87 (12.86)	12.87 (12.86)
CdSb	10	2	1.5	2.93	2.18	0.999	13.6 (14)	64.62 (64.9)
$CdAs_2$	4.5	2.3	1	0.515	0.645	0.609	44.86 (45)	18.66 (18.5)

TABLE 2. Geometrical Dimensions of Samples and Contacts.

Samples	Linear size a, mm	Thickness d, mm	Contact width $2c$, mm	$<f^{ex}>$	γ	L_{34}	σ_x, $Ohm^{-1} \cdot m^{-1}$	σ_y, $Ohm^{-1} \cdot m^{-1}$
Ge	14.7	0.6	1.05	1.145	1	0.235	12.85 (12.86)	12.85 (12.86)
CdSb	9.8	2	1.2	286	2.16	0.623	13.85 (14)	64.8 (64.9)
$CdAs_2$	4.3	2.3	0.85	0.3	0.645	0.185	44.9 (45)	18.6 (18.5)

MATERIALS AND METHODS

The experimental testing of technique 1 includes the preparation of square-shaped films cut off from Ge, CdSb and $CdAs_2$ crystals to fabricate ohmic contacts. The geometrical dimensions of the films and contacts are gathered in Table 1.

Using aurichlorohydric acid, gold contacts were deposited in the centre of the lateral surface of the Ge single crystal film. Indium solders were prepared for the anisotropic CdSb single crystal and solder POS-61 contacts – for the anisotropic $CdAs_2$ single crystal. Ohmicity of metal-semiconductor contacts was measured by the volt-ampere characteristics which turned to be linear within the current range of 20–50 mA.

After the preparation of the films and contacts, the direct current passed through contacts 1 and 2 according to technique 1 in 5 mA increments within the given range. The respective voltages U_{13} and U_{24} were then measured. A current source B5-44 was used for stabilization, and the voltage was measured with a high-resistance voltmeter V2-34. The electric conductivity tensor was determined in accordance with technique 1.

To confirm the reliability of the obtained results, ohmic contacts were formed on the same samples that occupied the whole width of their opposite sides. Then the electric conductivity was measured using a two-point probe method [9, 10].

Table 1 summarizes the reference figures of tensor components $\widehat{\sigma}$ calculated by the measurement results.

In the experimental testing of technique 2, we use the same samples for measuring the specific conductivity tensor components. The geometrical dimensions of the samples and contacts after polishing are gathered in Table 2.

As can be seen from Tables 1 and 2, the tensor values obtained by the proposed techniques are in good agreement with the reference values obtained by the two-point probe method. The difference is less than 3%.

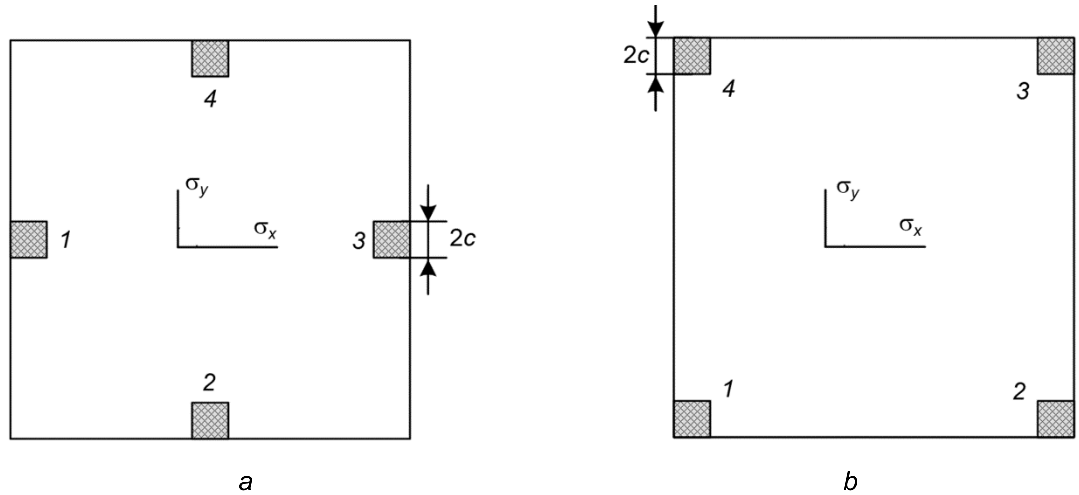

Fig. 5. The contact arrangement on the film surface: *a* – in the centre; *b* – at the corners.

MEASUREMENTS OF SPECIFIC CONDUCTIVITY OF SEMICONDUCTOR FILMS

Semiconductor films of various compositions are widely used in electronics, including anisotropic semiconductor films. In measuring the electric conductivity of thin films, it is suitable to arrange the contacts on the upper side as illustrated in Fig. 5. Let us assume that four square-shaped contact pads with the contact width $2c$ are positioned on the upper side of the semiconductor film. The electric fields produced by steady currents in anisotropic semiconductor films with the discussed geometry are studied in [14]. The authors suggest the equations for the calculation of the potential distribution between contacts *1–4*.

The error estimation of Eqs. (2)–(18) in relation to the contact layout presented in Fig. 5, the following values are found:

$$\varepsilon(2c/a)=\left|\frac{U_{24}^{(1)}-U_{24}^{(2)}}{U_{24}^{(2)}}\right|\cdot 100\ \%, \tag{19}$$

$$\delta(2c/a)=\left|\frac{U_{34}^{(1)}-U_{34}^{(2)}}{U_{34}^{(2)}}\right|\cdot 100\ \%, \tag{20}$$

where $U_{24}^{(1)}$ is the voltage between contacts *2* and *4* at I_{12} current flowing by the scheme in Fig. 1*a* and determined by Eqns (4), (5), respectively; $U_{34}^{(1)}$ is the voltage drop calculated by Eqns (11), (12) (Fig. 1*a*); $U_{24}^{(2)}$ (Fig. 5*a*) and $U_{34}^{(2)}$ (Fig. 5*b*) are calculated according to the analytic field distribution described in [14].

According to our calculations, the solution of Eq. (19) gives the value less than 5% in the case of only very thin films ($d/a < 0.001$) and unattainable dimensions of the contact pads ($2c/a < 0.001$). This is because one of the contacts in this case is the sensitive and current contact at a time. It is the current contacts that manifest the highest concentration of the field lines, and the effect from the shape and position of the contact pads is rather important in this case.

The results of the error estimation determined by Eq. (20) for the contact arrangement at the corners (Fig. 5*b*) are shown in Fig. 6. In this case, we see that Eqns (11)–(14) are applicable at different values of anisotropy. With the error less than 5%, it is enough to satisfy the condition of $d < 0.1a$, while the contact size must be $2c < 0.1a$ in most cases of the contact arrangement. It is not difficult to implement these conditions in practice.

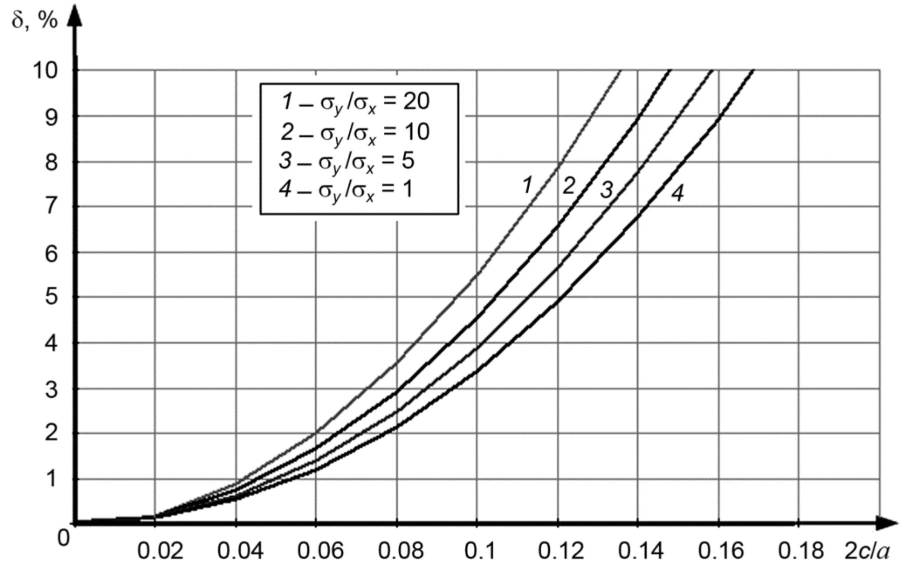

Fig. 6. Error estimation of Eqns (11), (12) for the voltage calculation between contacts *3* and *4* in Fig. 5.

Thus, we determine that practical measurements of the electric conductivity of anisotropic semiconductor films (Fig. 5*b*) can be based on the proposed technique 2 that utilizes the contact arrangement at the corners of the semiconductor film (Fig. 1*b*). Nevertheless, technique 1 (Fig. 1*a*) cannot be used for the electric conductivity measurements at the contact arrangement presented in Fig. 5*a.*

PRACTICAL RECOMMENDATIONS

The use of the proposed technique should satisfy the following requirements: 1) all the contacts must be ohmic; 2) measurements must be performed at the constant ambient temperature, uniform illuminance and low direct current in order to avoid strong heating of contacts and well-defined second-order effects.

The use of the Van der Pauw method in measurements of the electrical conductivity tensor of anisotropic semiconductor films was considered in a number of works [12, 15–17]. The equations presented in that works were rather cumbersome or obtained numerically. The expressions proposed in our work are simple, and series in Eqs. (3), (5), (12) and (14) are absolutely convergent and can be quickly calculated.

One of the virtues of the proposed technique of measuring the specific electrical conductivity tensor is the use of the small-size contact pads that do not affect the surface condition of the films. The contact layout formed in accordance with the Van der Pauw method allows using the same film to identify the hall constant without forming the new contacts [11]. Using, for example etching, these contacts can be easily removed from the crystal surface after measurements.

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